Debunking Debunking Economics

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1 Introduction.

These comments regard the mathematical exposition of the "new critiques" Steve Keen presents in his book *Debunking Economics*, available from the author at

http://www.debunking-economics.com/Maths/index.htm

The main arguments Keen makes are: (1) The theory of perfect competition is "mathematically flawed," (2) economists don't consider dynamics, and (3) the standard argument that monopolies are generally inefficient is invalid. I consider each in turn. Overall, these arguments are based on either a lack of familiarity with the literature, or conceptual errors, or both.

2 "Mathematical flaws" in the theory of perfect competition?

Part of the definition of a perfectly competitive market is that firms take prices as *parametric*: They decide on their actions as if those actions have no effect at all on prices. Keen argues that this is a "mathematical flaw" because no matter how small the firm is relative to the market, if it expands output price will fall. There is a lot of raw output from *Mathematica* provided to show this, but it's actually very trivial. A perfectly competitive firm choosing its output q to maximize profit solves

$$\max_q pq - c(q) \tag{1}$$

where p is prevailing price and $c(q)$ is the firm's cost function. This yields the familiar solution that price equals marginal cost. Keen's math simply notes that if the (inverse) market demand $p(\cdot)$ is smooth and downward sloping, then $dp(Q+q)/dq$ is not zero, holding the output of other firms Q constant. This is unremarkable, and very easy to show. It also follows that program (1) is not right for such markets, the firm "really" solves

$$\max_q p(Q+q)q - c(q) \tag{2}$$

where Q denotes the output of all other firms. It is easy to show the solutions of programs (1) and (2) will generally differ. It is also easy to show that the profits of any one firm are decreasing in the output of other firms from the equation above. Keen concludes:

> Where will this process of output reduction stop? With the standard assumptions of a downward sloping market demand curve and diminishing marginal productivity, it will stop at the same level of output as for a monopoly: where the marginal revenue for each firm equals its marginal cost.

Which is not correct. First, notice that the condition that profits are maximized when MR=MC is pretty general, it holds under both perfect competition and monopoly, for example. This condition also holds under many models of imperfect competition with strategic interactions, such as the one to be discussed below. It obviously isn't true that any solution characterized by MR=MC gives "the same level of output as for a monopoly." This is a rather remarkable misunderstanding of elementary theory which pervades Keen's analysis.

Second, it is effectively irrelevant whether or not firms ignore the effect of changes in own–output on price if they are small relative to the market. Let's flesh out an example in which firms do not "commit the fundamental mathematical error of setting a small quantity to zero." We can ask how much the "mistaken" solution and the solution when firms realize their own output affects price differ, and contrast both with the monopoly solution. Consider a simple, standard model often used to express such ideas (similar models can be found in any microeconomics textbook, and more involved discussion in narrower texts such as Tirole 1989). Suppose the market consists of n identical firms. Each firm has constant marginal costs c and market inverse demand is given by $P = a - bQ$. A few calculations shows the competitive solution is

$$Q^C = \frac{a-c}{b}. \tag{3}$$

This solution occurs if firms take prices as parametric, which is "wrong" for finite n. Suppose we relax that assumption and assume firms correctly calculate how their output affects price, and take competitors' output as given. A firm in this environment solves

$$\max_q p(Q+q)q - cq, \tag{4}$$

a few more lines of algebra yields an equilibrium total output

$$Q^N = \frac{n}{n+1}\left(\frac{a-c}{b}\right). \tag{5}$$

This is sometimes called the *Cournot–Nash* solution. Finally, we can also figure out what a monopolist who ran these n firms would do. A few more calculations reveals he would set total output

$$Q^M = \frac{a-c}{2b}. \tag{6}$$

Note that contrary to Keen, the Cournot–Nash solution and the monopoly solution are not the same. In fact, for reasonably large n, the Cournot–Nash solution and the competitive solution are all but indistinguishable.[1] Essentially, it's irrelevant for large n whether we go to the trouble of calculating what happens when each firm has a little bit of market power, or simply assume firms take prices as parametric. The monopoly output, conversely, is exactly one half the competitive output in this simple model. Allowing that firms have market power does not imply that the monopoly solution results.

Second, as n becomes large, $n/(n+1)$ approaches unity. The Cournot–Nash solution approaches the competitive solution. The competitive solution is the *limit* of this form of imperfect competition. Put another way, one manner of rigorously deriving perfect competition is considering what happens as the number of firms in an imperfectly competitive market is allowed to grow without bound.

Keen concludes:

> The theory of perfect competition is therefore fundamentally flawed. Far from being an ideal market structure by which all other market types should be judged, it is an illusion built upon fundamental mathematical fallacies

[1] Note the ratio of output under competition to that under Cournot is $(n+1)/n$. For example, in a market with but 10 firms, the competitive output is only about 10% higher than that under imperfect competition. With 10,000 firms, the difference is one percent of one percent.

It is not true that the theory is mathematically flawed. We have seen a simple example of one manner of rigorously obtaining a mathematical model which is not subject to the alleged flaw is to consider limits of models of imperfect competition. Another approach to rigorously deriving perfect competition is to assume there exists a continuum of firms. In that case, it is literally true that any firm can change its output without changing price, even when the market demand is smooth and downward–sloping. Aumann (1964) is a standard reference.

Equally importantly, we noted that even when the number of firms is finite and countable, imperfect competition amongst a surprisingly small number of firms can yield outcomes which are indistinguishable from the abstract perfectly competitive case. This isn't a result which hinges on the assumptions in the simple model we considered. When Fred Smith is deciding how intensively to farm his 500 acres of wheat, he need not consider how variations in his output will affect the world price of wheat. Keen is correct that, formally, he ought to: Perhaps if he doubles his output the price will fall by a few millions of a cent per bushel (in expectation?). But any differences between his actions if he does or does not ignore this tiny price change are not worth considering.[2]

3 Economists don't consider dynamics?

Keen alleges economists don't consider dynamic models in any substantive sense:

> It is feasible to see Sraffa's critique as simply an attempt to take seriously the limitations which Jevons, Walras, Marshall and Clarke all acknowledged were endemic in using static methods to analyse what are clearly dynamic problems. Their defence for the use of static tools was the inherent difficulty of dynamic analysis,

[2]Keen makes much of the obvious fact that $dP(Q)/d(Q) = dP(Q)/dq$, in his notation. In this case, that equation simply means that Farmer Smith's extra bushel has the same miniscule effect on price as an extra bushel from anyone (or everyone) else. The quote from the Lipsey and Chrystal introductory textbook is correct; the authors are not, as Keen strangely alleges, copping an "attitude" that the slope of the market demand curve is zero. Rather, they are pointing out that the firm's optimal decision is effectively the same whether or not it ignores the fact that market demand is sloped, provided q/Q is sufficiently small.

> and the absence of suitable tools. No such defence is available to modern economics, since dynamics is now a far more developed field of analysis (in sciences other than economics), and so many tools exist to analyse dynamic systems dynamically. We can begin this process of recasting economics as a dynamic science by taking Sraffa's critique to heart and drop altogether the neoclassical treatment of time.

Let us put this bizarre claim to rest immediately, then discuss Keen's example. Dynamic models are very commonly employed in modern economic research. Almost all of modern macroeconomics is based on such models, and huge swathes of theory in other fields are similarly dynamic. Popular undergraduate level references on methods used to solve such models include Kamien and Schwartz (1981) and Chiang (1999). Such methods have been quite common in economics for the last three decades, although famous early examples date back to the twenties, see for example the early contributions of Ramsey (1928), Hotelling (1931), Bellman (1954) and Dorfman (1969). Dynamic models are hardly obscure in modern economic research, much less absent altogether as Keen alleges.

Keen gives an example of a profit–maximizing firm in a dynamic environment in order to demonstrate what he considers the failing of mainstream theory. The problem is neither set up nor solved correctly. Keen considers a firm which obtains profits at time t of

$$\Pi(q(t),t) = P(q(t),t)q(t) - TC(q(t),t). \tag{7}$$

Keen assumes the firm sets output at time t, denoted $q(t)$, to maximize the *rate of change* of profit at point t. But it makes no sense to assert firms maximize the instantaneous rate of growth of profit. Suppose, for instance, that this rate can be made arbitrarily high over an arbitrarily short period, but that doing so comes at great cost in the future. For example, a discrete increase of \$1 at point of time t makes this rate infinite. Suppose that \$1 increase in profits comes at a cost of losing \$1,000,000,000,000,000 every moment until the end of time. If the firm acted to solve Keen's optimization problem, it would set out to earn that dollar.

So what would a profit–maximizing firm do in such a dynamic environment? It would maximize the *discounted value* of profits. That is, it would weigh off revenues and costs over time to make it's present value as high as possible. A firm in a continuous time, deterministic environment might be modeled as solving

$$\max_{q(\cdot)} \int_0^\infty \pi_t(q(t))e^{-rt}dt$$

subject to the relevant laws of motion and boundary conditions, where π_t is the instantaneous rate of profits at time t, which generally depends on *the function* $q(t)$. Profits at any point in time generally depend not only on its rate of output at that point, but also on rate of output at all other times.[3]

It is important to note that this argument is not a quibble over objective functions. A firm in a (continuous) dynamic environment sets the *function* $q(\cdot)$ to maximize a *functional*, one example of which is the present value of profits. The firm chooses output at every point in time, not one point in time as in Keen's analysis. Generally, decisions at any point in time depend not only on the tradeoff between instantaneous costs and benefits, but also on how decisions "today" will affect the firm in the future. It is ironic that Keen's method actually assumes away any point to solving the dynamic problem; if the firm's choice of output at time t does not affect its situation before or after t, the problem is essentially static.

Methods for static optimization cannot be applied to this class of problem because the firm chooses a function, not a number or countable set of numbers.[4] A common method of solving this often difficult class of problem is called *optimal control*. Details can be found in either of the texts mentioned above. It is worth noting that along an (interior) solution path, the firm sets marginal revenue equal to marginal cost in the relevant sense, contra Keen. But these expressions are not evaluated in terms of their effects on instantaneous profits, but rather in terms of changes in the present value of profits along the solution path.

[3]For example, in models of *learning–by–doing,* costs at time T might be a decreasing function of cumulative past output $\int_0^T q(s)ds$. Increasing output at that point in time is less costly than the instantaneous increased expenditure in the sense that the firm places itself in a better situation in the future.

[4]Notice this difficulty arises in Keen's analysis, even with his essentially static objective. In equation (18), what is the firm choosing? Keen implicitly assumes the firm chooses $q(t)$ (the single number: quantity at time t), but $dq(t)/dt$ also a choice variable, not something imposed on the firm. If it is a choice variable, Keen's firm will set any $q(t)$ such that MR>MC and allow the objective to become arbitrarily large by allowing $dq(t)/dt$ to become arbitrarily large. Of course, a correct solution for any reasonable objective involves setting the *function* $q(t)$, or *equivalently* its rate of change at every time.

4 The standard comparison of monopoly versus competition is invalid?

Keen considers total output from n firms $nf(x)$, and an "output function" for a monopolist $g(\cdot)$. Keen shows that if $nf(x) = g(nx)\,\forall n$, then $g(\cdot)$ must be linear. A much simpler proof than the one Keen gives is to simply observe that the left–hand side is linear in n; therefore the right–hand side must also be linear in n, and $g(nx)$ is only linear in n if $g(\cdot)$ is linear. This is obvious and does not imply Keen's conclusions, unless the textbook argument is misinterpreted.

Consider a market comprised of n firms. Suppose the i^{th} firm has cost function $c_i(q_i)$ and that marginal costs are weakly increasing in output. What mathematical relation characterizes an efficient allocation of output across firms? That is, if total output is to be Q, how should output be distributed across firms to minimize costs? The relevant program is

$$\min_q \quad \sum_i c_i(q_i) \tag{8}$$

$$s.t. \quad \sum_i q_i = Q. \tag{9}$$

It is simple to show that the solution is characterized by setting marginal costs equal across firms. Denote the solution vector q^*. The cost of producing Q units of output is $C(Q) = \sum_i c_i(q_i^*)$.

Under perfect competition, each firm sets output to equate marginal costs and price. Since each firm faces the same price, it follows that marginal cost is equated across firms in the competitive solution. Therefore, the equations solved to find q^* above also apply to the competitive allocation, so the competitive industry spends $C(Q)$ to produce Q units of output. The competitive industry is efficient in the sense defined above.

A profit–maximizing monopolist solves

$$\max_{\{q_i\}} P(\sum_i q_i)(\sum_i q_i) - \sum_i c_i(q_i) \tag{10}$$

The solution is partially characterized by the condition that marginal costs should be equated across firms, as before. This is simply because maximizing profits implies minimizing costs, so the monopolist sets out to enact that which happens as an emergent outcome in the decentralized market. That is, the monopolist spends $C(Q)$ to produce Q units of output. The monopolist will generally set a different total output than that which occurs under competition, which is where the standard argument why monopolies are inefficient arises. But we have shown that for any given output, either market structure yields precisely the same costs. It follows that marginal costs are also identical, and therefore that the monopolist's marginal cost curve and the market supply curve are one and the same.

Keen's argument is therefore incorrect. It is incorrect because adding x more workers to the competitive market would not generally add $f(x)$ to total output.[5] Those x workers would be "split" across firms because of diminishing returns in the manner described above, and would add more than $f(x)$ to total output. We have seen above that a monopolist would "split" these additional workers across plants in exactly the same manner, and would therefore receive exactly the same increase in output. Keen's derivations simply show if and only if $f(\cdot)$ is linear, then this "splitting" is irrelevant, and either a monopolist or the competitive industry would produce additional output proportional to x.

It is worth noting that Keen's argument is correct if "monopolize the industry" is misinterpreted. If we interpret that phrase as "give one firm with technology $f(\cdot)$ a monopoly," then it is true that the monopolist's costs will differ from those under competition unless the technology exhibits constant returns. This result follows from the observation that, for concave $f(\cdot)$, $f(nx) \leq nf(x)$, with equality only when $f(\cdot)$ is linear. But this is of course not what is meant by "monopolize the industry." For example, by "monopolize the U.S. wheat market" we mean "the entire U.S. wheat industry is operated to maximize its total profits." We don't mean, "Fred Smith's 500 acre farm in southeast Nebraska is given a monopoly on production of U.S. wheat."

[5] Note that, technically, Keen is not varying number of workers, he is varying number of firms. I am interpreting his argument as charitably as possible since he is not, as he claims, comparing the same industry across two market structures, he is changing the technology in the industry. Another manner of objecting is to note that the monopolist's technology generally depends on n directly, so writing $g(x)$ rather than $g(x, n)$ is incorrect.

5 References.

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